AP Rev. WS #7 - Calculator required for several of these.

1. $f'(x) = (\cos e^{-x})(-e^{-x})$ E

2. $y' = 1 - \sin x$

slope @ $x = 0 \rightarrow 1 - \sin 0$

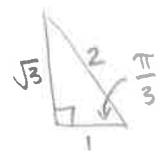
$m = 1$

tangent through $(0,1)$

$y = x + 1$ B

3. $f'(x) = 2\sec^2 2x$

$f'(\frac{\pi}{6}) = 2(\sec\frac{\pi}{3})^2$



$= 2(\frac{2}{1})^2$

$= 8$ E

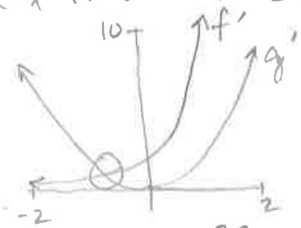
4. Parallel tangents $\rightarrow$ same slope

so $f'(x) = g'(x)$

$6e^{2x} = 18x^2$

$e^{2x} = 3x^2$

graph & find intersection.

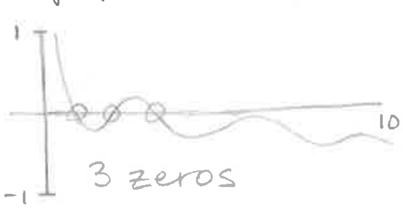


$x \approx -0.391$ C

5. Critical numbers where $f' = 0$ or is undefined.

f' is undef. @ $x = 0$

graph f' on $(0,10)$ & count zeros.



since $x = 0$ is not on the open interval $(0,10)$, there are 3 critical #'s B

6. $f'(x) = \dfrac{1 - 3e^{-3x}}{x + 4 + e^{-3x}}$

$f'(0) = \dfrac{1 - 3(\frac{1}{e^0})}{0 + 4 + \frac{1}{e^0}}$

$= -\frac{2}{5}$ A

7. $\dfrac{dV}{dt} = kV$ E

8. $x^2 \cdot 2\cos 2x + 2x\sin 2x$

$2x(x\cos 2x + \sin 2x)$

E

9. Given 2 pts. on the tangent

$(1,7)$ & $(-2,-2)$ $m = \dfrac{7+2}{1+2}$

$m = 3$ C

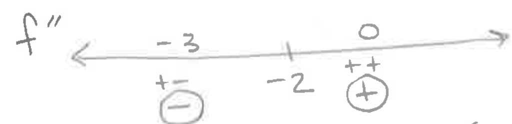
10. $f'(x) = 2xe^x + 2e^x$

$f''(x) = 2xe^x + 2e^x + 2e^x$

$0 = 2xe^x + 4e^x$

$0 = 2e^x(x+2)$

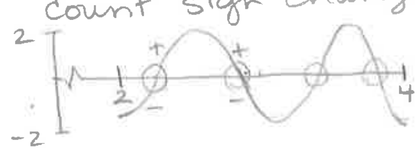
$2e^x \neq 0$ $\searrow$ $x = -2$



concave down on $(-\infty, -2)$

A

11. $0 = \sin(x^2 + 1)$ graph and count sign changes on $(2,4)$



4 relative extrema

D

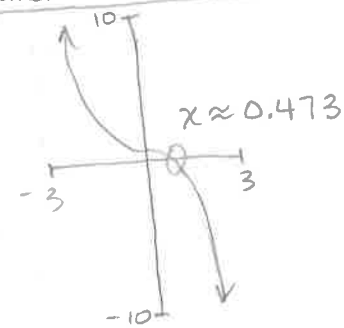
12. $f''(x) = \dfrac{(1+x+x^3)\cdot\frac{1}{2\sqrt{x}} - \sqrt{x}(1+3x^2)\left(\frac{2\sqrt{x}}{2\sqrt{x}}\right)}{(1+x+x^3)^2}$

$= \dfrac{1+x+x^3-2x(1+3x^2)}{2\sqrt{x}(1+x+x^3)^2}$

$f''(x) = \dfrac{-5x^3-x+1}{2\sqrt{x}(1+x+x^3)^2} \longrightarrow -5x^3-x+1=0$ graph & find zeros.

always +
$x \neq 0$

Numerator of f''



f'': 0.1, 1; $+$ | ≈ 0.473 | $-$

(B)

13a) $a(t) = -(t+1)\cdot t\cos\left(\frac{t^2}{2}\right) + (-1)\sin\left(\frac{t^2}{2}\right)$

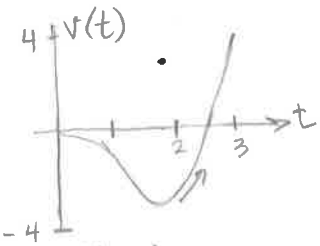
$a(t) = (-t^2-t)\cos\left(\frac{t^2}{2}\right) - \sin\left(\frac{t^2}{2}\right)$

$a(2) = (-6)\cos(2) - \sin(2)$

$\boxed{a(2) \approx 1.588}$ } acc +

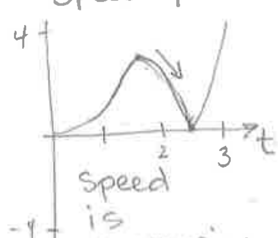
$v(2) = -3\sin(2) \approx -2.73$ } vel. −

Since $a(2)$ & $v(2)$ have opposite signs, the particle's speed is decreasing.



velocity is negative and getting closer to zero @ t=0
Speed is decreasing.

Speed = $|v(t)|$



Speed is decreasing @ $t=2$

b) The particle changes direction when velocity changes sign. From the graph of $v(t)$ above, velocity goes from − to + @ $\boxed{t \approx 2.507}$

Algebraic Method: $v(t) = 0$

$-(t+1) = 0$ $\quad\quad$ $\sin\left(\frac{t^2}{2}\right) = 0$

$t = -1$ $\quad\quad$ $\frac{t^2}{2} = \pi n$

Not on interval

$t^2 = 2\pi n$

$t = \pm\sqrt{2\pi n}$

critical numbers

on the interval $0 < t < 3$

$t = \sqrt{2\pi}$

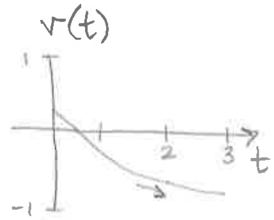
v: 1, 2.7; $-$ | $\sqrt{2\pi}$ | $+$

≈ 2.507

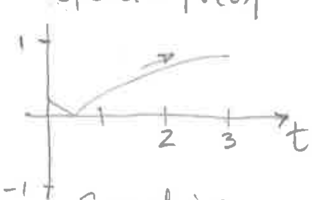
14a) $a(t) = 0 - \frac{e^t}{1+e^{2t}}$

$$\boxed{a(2) = -\frac{e^2}{1+e^4} \approx -0.133}$$

b) $a(2) \approx -0.133$
$v(2) \approx -0.436$ } since both vel & acc are negative, the particle's speed is increasing @ $t=2$



velocity is getting further away from zero, so speed is increasing @ $t=2$



Speed is increasing @ $t=2$

c) Maximum height when $v(t)=0$ and goes from + to −
$1 - \tan^{-1}(e^t) = 0$
Graph $v(t)$ & find zeros.
$t \approx 0.443$ and graph goes from + to − there

15a) Increasing @ $t=6$ means $R(6)$ is positive.
$R(6) = 5\sqrt{6}\cos\left(\frac{6}{5}\right) \approx 4.438$ Positive rate of change, so the number of mosquitoes is increasing.

b) # of mosquitoes increasing @ an increasing rate means R' +
decreasing rate mean R' −

$R'(t) = 5\sqrt{t}\left(-\sin\frac{t}{5}\right)\left(\frac{1}{\cancel{5}}\right) + \frac{5}{2\sqrt{t}}\cos\frac{t}{5}$

$R'(t) = -\sqrt{t}\sin\frac{t}{5} + \frac{5}{2\sqrt{t}}\cos\frac{t}{5}$

$R'(6) = -\sqrt{6}\sin\frac{6}{5} + \frac{5}{2\sqrt{6}}\cos\frac{6}{5}$

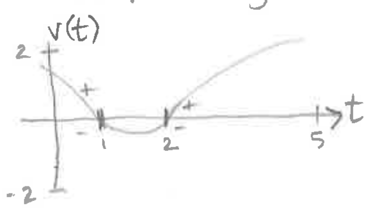
$R'(6) \approx -0.064$, so the number of mosquitoes is increasing at a decreasing rate.

16a) $a(t) = \frac{2t-3}{t^2-3t+3}$

$a(4) = \frac{5}{16-12+3}$

$$\boxed{a(4) = \frac{5}{7}}$$

b) Particle changes direction when $v(t)$ changes sign.



$v(t)$ goes from + to − at $\boxed{t=1}$ and from − to + at $\boxed{t=2}$

Particle travels to the left when $v(t)$ is negative.

$$\boxed{1 < t < 2}$$